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Performance of Advanced Stock Price Models when it becomes Exotic: an Empirical Study

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Abstract

We calibrate several advanced stock price models to a time series of real market data of European options on the DAX. Via a Monte Carlo simulation, we price barrier down-and-out call options for all models and compare the modeled prices to given real market data of the barrier options. The Bates model reproduces barrier option prices very well. The BNS model overvalues and Lévy models with stochastic time-change and leverage undervalue the exotic options. The Heston model and a local volatility model undervalue the barrier option prices by about 5-6%. A heuristic analysis suggests that the different degree of fluctuation of the random paths of the models are responsible of producing different prices for the barrier options. Higher margins or additional risks like liquidity, calibration or model risk might economically explain why many advanced models undervalue barrier options.

JEL classification: G13
Keywords: Barrier options, empirical performance, advanced stock price models, stochastic volatility for Lévy processes.

1 Introduction

In this article we analyze advanced stock price models empirically. There is an endless list of advanced stock price models generalizing the Black-Scholes model and being able to capture many stylized facts typically observed in financial time series like fat-tail behavior of log-returns, volatility clustering and negative correlation between volatility and stock price movements known as the *leverage effect*, see Cont (2001) for a more extended list of stylized facts of financial times series.

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There are several studies in literature showing that many advanced stock price models can be calibrated very well to plain vanilla option data, in the sense that different models lead to almost identical plain vanilla prices. However, the calibrated advanced stock price models may lead to very different exotic option prices. This is known as *model risk*.

We calibrate six advanced stock price models and a local volatility model to a time series of real market data of European options on the DAX, a German blue chip index. Via a Monte Carlo simulation, we price barrier down-and-out call options for all models and compare the modeled prices to given real market data of the barrier options. A heuristic analysis of the six advanced stock price models is carried out to investigate why the models predict different prices of the barrier options.

There are two contributions: this is the first study which focus only on models describing the volatility by a stochastic process and incorporating a leverage effect. To the best of our knowledge, this is the first time exotic option prices are simulated under Lévy processes with stochastic time-change and leverage. In contrast to former studies, we also compare the simulated prices to real market data of equity barrier options. Hence for our particular data set, we are able to decide which model reproduces barrier option data best.

The Models

Let us recall some models often applied in literature. Local volatility models (LV) replace the constant volatility of the Black-Scholes (BS) model by a deterministic function of both time and current underlying level, see Dupire (1994). A special case is the constant elasticity of variance model introduced by Cox (1975).

The Heston (HES) model, see Heston (1993), replace the volatility of the BS model by a mean-reverting stochastic process, the square root process of Cox, Ingersoll, and Ross (1985), abbreviated by CIR. The volatility is allowed to be correlated with the uncertainty driving the log-returns and hence the HES model is able to model both stochastic volatility and the leverage effect. Bates (1996) generalized the Heston model incorporating the possibility of jumps of the stock price. The resulting model is abbreviated by HESJ.

Barndorff-Nielsen and Shephard (2001) developed the BNS model replacing the constant volatility of the BS model by an Ornstein-Uhlenbeck (OU) process. We use the classical example of the Gamma-OU process. It is possible to include a leverage effect into the BNS model.

One could as well replace the Brownian motion of the BS model by a more flexible Lévy process. A prominent example is the variance gamma (VG) model developed by Madan et al. (1998). Pure Lévy models are quite flexible and are able to describe skewness and fat tail behavior of log-returns but are stationary over time.

Lévy models with stochastic time-change have been described in detail by Schoutens (2003) including algorithms to simulate such processes. As in the BNS model, it is possible to allow the stochastic time change to directly effect the log-returns. We call such models *Lévy models with stochastic time-change and leverage*, see Carr et al. (2003).

In this article we take the HES, HESJ and the BNS model and three Lévy models. As Lévy processes we choose the Normal Inverse Gaussian (NIG) process, see Barndorff-Nielsen (1997), the VG process and the Merton jump-diffusion model (MJD), see Merton (1976), and subordinate them by a random time-change modeled by the CIR process. Other choices are possible, see Carr et al. (2003). The three resulting stock price models are abbreviated by NIG_CIRL, VG_CIRL and MJD_CIRL, where 'L' stands for leverage.

We chose those six models, because all models are able to capture stochastic volatility and the leverage effect and are flexible enough to model plain vanilla options reasonably well, see Schoutens et al. (2003). They are further mathematically tractable, can be calibrated relatively fast to real market data and it is straightforward to implement the models in order to perform a Monte Carlo simulation. We also include the LV model as it is a standard model in industry.

Literature Review

Often, a set of prices of European plain vanilla options are given and financial models are calibrated to market data of plain vanilla options and are then used to price exotic options or to construct trading strategies to replicate plain vanilla or exotic options as good as possible. Most of the presented models can be calibrated very well to plain vanilla option data in the sense that different models lead to almost identical plain vanilla option prices. The question is whether prices under different models of an exotic option are also approximately equal. The clear answer is *no* as the following three studies show.

i) Hirsa et al. (2003) calibrated the VG model, a LV model, the constant elasticity of variance model and the VG model with stochastic time-change (but without leverage) to plain vanilla options on the S&P 500 index and priced barrier options under the different models. They concluded: "regardless of the closeness of the vanilla fits to different models, prices of up-and-out call options (a simple case of exotic options) differ noticeably when different stochastic processes are used to calibrate the vanilla options surface".

ii) Schoutens et al. (2003) calibrated the Heston model (with and without jumps), the BNS model and various Lévy processes with stochastic time change (but without leverage) to plain vanilla option data on the Eurostoxx 50 index and used the calibrated models to price various exotic options among them different types of barrier options. They concluded that all those models can be calibrated almost perfectly to plain vanilla option data but the resulting exotic option price can differ significantly.

iii) Jessen and Poulsen (2013) calibrated the BS model, the constant elasticity of variance model, the Heston model (with and without jumps), the Merton jump-diffusion model, the VG model and the VG model with stochastic time-change (but without leverage) to plain vanilla options on the USD/EUR exchange rate and priced different types of foreign exchange barrier options. They concluded: "Models may produce very similar prices of plain vanilla options yet differ markedly for exotic options."

In contrast to the two former studies i) and ii) who only worked with real data of plain vanilla options, Jessen and Poulsen (2013) compared the modeled prices of barrier options to given market data of the barrier options. For their particular data, the constant elasticity of variance model best explained the market data of barrier options, leading to an average relative error of just 0.13%. The Heston model undervalued the barrier options by 3.48% on average. The Heston model with jumps priced barrier options significantly worse than the Heston model with an average absolute errors of 24.7%.

What can we add to the studies i)−iii)? To the best of our knowledge only Jessen and Poulsen (2013) compared model prices of exotic options to real market foreign-exchange data. We repeat that study for equity data. In contrast to former studies, we will focus on models incorporating both stochastic volatility and the leverage effect.

Default Risk

Both the plain vanilla and the barrier options are issued by financial institutions which might default. In this study we do not model the credit risk of the issuers. We argue that we can neglect the default risk of the issuers because the issuers have a very high creditworthiness. By this argumentation, we follow Chen and Kensinger (1990), Chen and Sears (1990), Wasserfallen and Schenk (1996), Burth et al. (2001) and Henderson and Pearson (2011) among others. Furthermore, the price of an option emitted by some issuer is directly influenced by the issuer's default risk. Therefore the calibrated models implicitly contain the credit risk already. For a direct approach to model the credit risk of the issuer of derivatives, see Hull and White (1995).

Contents

This article is structured as follows. In Section 2 we describe in detail the data set used in this empirical study. Sections 3 comments on the calibration and pricing procedure via Monte Carlo. Section 4 compares the six models, when applied to exotic option data and offers some explanation *why* some models overvalue and other models undervalue barrier options. Section 5 concludes.

2 Market Data

2.1 Plain Vanilla Options

We look at a time series from 05/07/2017 till 21/08/2017, which contain 34 trading days. At each trading day, there are three timepoints, namely "morning"

(10am-10:30am), “midday” (1pm-1:30pm) and “afternoon” (4pm-4:30pm) on which prices of in total about 471.000 European plain vanilla put and call options with maturities ranging from $0-3$ years and moneyness ranging from $0.5-1.0$ are available on the DAX, a blue chip stock market index consisting of the 30 major German companies.

We follow the methodology applied for the volatility index (VIX) by the Chicago board of exchange, see CBOE (2018), and only use out-of-the-money options for calibration, see also Carr and Wu (2009).

Those options are issued by the financial institutions Commerzbank, UBS and UniCredit, which usually also act as a market maker. The options are listed on different stock exchanges in Germany, mainly on Frankfurt stock exchange and Stuttgart stock exchange. They can also be traded over the counter, directly with the issuer. The absolute average bid-ask spread over all products of all banks is 0.03 EUR. The average relative spread is 5%.

2.2 Exotic Options

We obtained in total 303.000 bid and ask quotes of down-and-out barrier (DOB) call-options for the same time series consisting of 102 timepoints in the period from 05/07/2017 till 21/08/2017 and issued by the same issuers as described in the previous section. The payoff of such option with strike K and maturity T is

$$\mathrm{DOB}(K,T) = \begin{cases} \max(S_T - K, 0) & , \inf_{0\le t\le T} S_t > K \\ 0 & , \text{otherwise.} \end{cases}$$

The strike and the barrier are equal. The process $(S_t)_{t\ge 0}$ describes the stock price under an equivalent martingale measure. A call option with the same maturity T and strike K is called *the corresponding plain vanilla option.* If the barrier is hit before maturity, the DOB option becomes worthless, otherwise it has the same payoff as the corresponding plain vanilla option. It is clear that the corresponding plain vanilla option is always in-the-money otherwise the barrier would be knocked-out.

For each exotic option, we compute the price of the corresponding plain vanilla option using an implied volatility surface which we obtain from the plain vanilla data set, see Section 2.1. We focus on all exotic options which are “exotic” enough, i.e. whose exotic prices are smaller than 0.75 times the corresponding plain vanilla prices. This essentially means removing all exotic options whose corresponding plain vanilla options are deep in-the-money. We are then left with about twenty thousands exotic options with maturities ranging from a few days to half a year. The moneyness of the corresponding plain vanilla options lay between 1.01 and 1.07.

Due to our selection-procedure we focus on exotic options whose barriers lay slightly below the underlying. We therefore face a different set of exotic options at each timepoint: as the underlying is changing over time, some barrier option might be knock-out, if the underlying decreases, and hence disappear from the data set. Or, if the underlying increases, some barrier options might move too far in-the-money, and are also removed from the data set because they are not exotic enough any more and are filtered by our selection procedure. Therefore the set of identification numbers of the exotic options and the strikes differ over time. But the maturities and the moneyness of the corresponding plain vanilla options are more or less stable over time. 99.9% of the absolute bid-ask spreads of all exotic options are smaller or equal than 0.02 EUR.

3 Methodology

3.1 Calibration

For each issuer and at each timepoint, we calibrate the models BNS, HES, HESJ, MJD_CIRL, NIG_CIRL and VG_CIRL to prices of plain vanilla options by minimizing the mean-square error between market prices and model prices. The LV model is calibrated by Dupire's formula. We obtain for each model a time series of parameters. All models can be fitted very well to real market data. This is in line with the results of Schoutens et al. (2003).

The practice of recalibrating the models at each timepoint is difficult to justify economically but it is an industrial standard to ensure that a model price liquid plain vanilla options as good as possible, see e.g. Jessen and Poulsen (2013). Over time a financial market changes, new information arrives etc. which leads to the need of recalibrating the models.

3.2 Pricing via Monte-Carlo

For each issuer, each timepoint and each model, we take the parameters obtained by calibrating the various models to plain vanilla option market prices, see Section 3.1, and price all available exotic barrier options via Monte-Carlo simulations. For each options we use $M = 200,000$ simulations and a timestep of $\delta = 4.0 \cdot 10^{-5}$ business years, which corresponds to a grid-size of about five minutes. Such a narrow grid-size is necessary, to keep the discretion error small. The price of an option whose barrier is very close to the underlying reacts quite sensitive to the number of time-steps chosen to discretize the underlying. We use variance reduction techniques by control-variates. The simulation of jump-diffusion models are standard, see for example Glasserman (2013). Simulation techniques for Lévy processes with stochastic time-change and the BNS model can be found e.g. in Schoutens (2003).

4 Pricing Ability of The Models

Figure 1 shows the relative differences between market and model prices of all exotic options for the various models with maturities between three and six months. On average HES undervalues exotic options by about 5.5%. HESJ explains exotic option market data best, but still undervalues options by about 1%. The BNS model has the greatest bias overvaluing the options by about

19% on average. The Lévy models MJD_CIRL, NIG_CIRL and VG_CIRL undervalue the barrier options by about 13% on average. The LV model undervalues the barrier options by about 5.9%. A similar Figure could be produced for barrier options with maturities up to three months.

If financial players would use the LV model for pricing purposes, this would imply they charge about 5-6% extra premium compared to more simple plain vanilla options. This premium could cover their margin as well as extra charges for additional risks, like model risk, calibration risk and liquidity risk. Similar premium estimates could be made on the basis of the other models like the Heston model.

In this context, Stoimenov and Wilkens (2005) observed that "all types of equity-linked structured products are, on average, priced above their theoretical values and thus favor the issuing institution [...]. In general, more complex products incorporate higher implicit premiums", see also Wilkens and Stoimenov (2007) and Baule and Tallau (2011).

The patterns are independent of the issuer of the exotic options and are similar also under different market environments: while the first half of the analyses timepoints correspond to a rather calm market environment: the VDAX_NEW, a volatility index for the DAX, get as low as 12.2%, in the second half, uncertainty measured by the volatility index raises at its top to 18%, which is a difference of 50% to its lowest level. In mid-August 2017, the North Korea crisis escalated rhetorically, which explains the rise of the volatility index economically.

MJD_CIRL is less stable than the other models. On August, 11th in the morning, there is a big peak at timepoint 82 in Figure 1 demonstrating that MJD_CIRL is undervaluing the barrier options by up to 60%.

Usually, we calibrated the models using a global stochastic optimizer, see Storn and Price (1997). However, calibrating MJD_CIRL at timepoint 82 instead using the Nelder-Mead algorithm, see Nelder and Mead (1965), to search for an optimum and choosing the optimal parameter set of the previous timepoint 81 as starting point, leads to a much better result: the relative difference between model and market prices shrinks sharply as shown by a point in Figure 1. Calibrating MJD_CIRL is less robust than calibrating the other models.

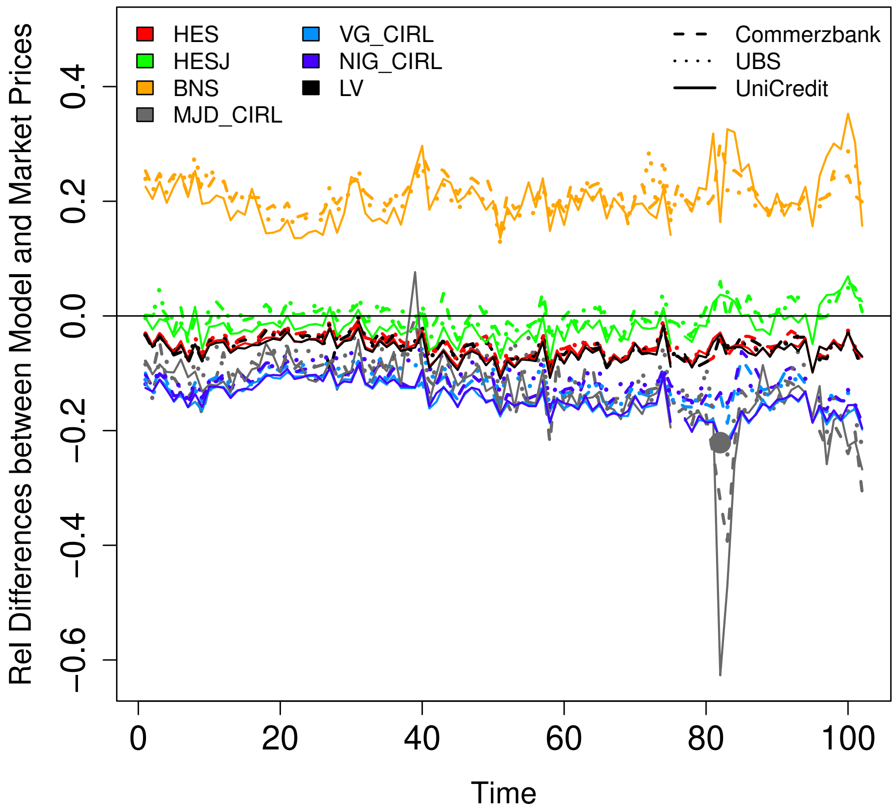

Figure 1: Relative differences for maturities more than 3 months. The point at timepoint 82 corresponds to another parameter set for the MJD_CIRL model

4.1 Path Characteristics

In this section, we attempt to provide some characteristics of the path-behavior of the various models under the risk-neutral measure. That helps to understand the models and might (partially) answer the question why some models overvalue and other models undervalue the barrier option data set from Section 2.2.

For a stochastic process $S_{t\in[0,1]}$ describing a stock price we define some characteristics as follows. All characteristics refer to the interval $[0, 1]$, i.e. to the period of one year.

$$\begin{array}{rcl} h &=& \text{Probability the process hits } 0.95S_0 \text{ at least once,} \\ \sigma &=& \sqrt{\mathrm{Var}(S_1)}, \\ J_{\log} &=& \text{Expected number of times the absolute value} \\ && \text{of the log-returns jump by more than 5\%,} \\ \mathrm{ARV} &=& \text{Average realized variance (defined below),} \\ U &=& \text{Expected number of times the processes crosses } [0.98S_0, 1.02S_0]. \end{array}$$

Model	h	σ	$J_{\log}$	ARV	U
BNS	0.66	0.26	1.32	0.063	1.22
HESJ	0.72	0.27	0.03	0.073	1.47
HES	0.74	0.19	0.00	0.032	1.59
MJD_CIRL	0.83	0.22	0.15	0.068	2.35
NIG_CIRL	0.84	0.24	0.91	0.079	2.47
VG_CIRL	0.86	0.26	0.69	0.096	2.82

Table 1: Some characteristics of the path-behaviour of the various models. The table is ordered by column h. Each model is simulated $M = 200,000$ times on the time interval $[0, 1]$ using a step-size of $\delta = 4.0 \cdot 10^{-5}$ business years or equivalently using $N = 25,000$ time-steps. We assume $S_0 = 100$ and $r = 0$ for all models.

We estimate the values by a Monte Carlo simulation for the six models based on the average calibrated parameters. The average is taken over all timepoints and issuers. Those parameters do not belong to any particular set of plain vanilla options, nevertheless the computed characteristics shown in Table 1 are similar when choosing the parameter sets of a particular timepoint and issuer. Note that those characteristics are computed under an equivalent martingale measure and might look very different under the physically measure.

Clearly, the probability of hitting the barrier directly influences the price of a barrier option. The value h is comparatively low for the BNS model and high for the Lévy models. Indeed in Section 4, we show that the BNS model overvalues and the Lévy models undervalue the barrier options. But the probability of hitting the barrier can neither be explained very well by the standard deviation nor by the expected number of (big) jumps of the processes: the standard deviations are approximately in the same range for all models. For instance the BNS model and the VG_CIRL model have the same standard deviation at time $t = 1$ but significantly different probabilities of hitting the barrier. The number of expected jumps are particularly high for the BNS *and* the Lévy models.

We also compute for the six models the average realized variance (ARV): we discretize the interval $[0, 1]$ by $N = 25,000$ time-steps and for a simulated path we add up the squared log-returns between two successive time-points. We repeat this $M = 200,000$ times and take the average. See for example

Barndorff-Nielsen and Shephard (2002) for a precise definition of ARV and its relation to quadratic variation of semimartingales. (We also computed the ARV based on daily returns, but the result does not differ much).

It turns out that the ARV of the Lévy models NIG_CIRL and VG_CIRL are significantly higher than the ARV of the BNS model, which indicates that the Lévy models have a higher path-fluctuation compared to the BNS model. But the ARV of the BNS model is about twice as large than the ARV of the HES model.

The low ARV of the HES model might be explained by the fact that HES is a continuous model without any jumps. However, the probability of hitting the barrier is considerable higher for the HES than for the BNS model and we conclude that the ARV does not really help to understand the different pricing behavior of the various models.

The characteristic U seems to provide some explanation: U measures the expected number of up-crossings of some interval around the starting point of the stock price process. U indicates how often a stock price process changes its direction. The higher U, the higher the fluctuation of the sample random paths under the risk-neutral measure.

Even though the BNS model jumps quite often (and each jump is directed downwards increasing the probability of hitting the barrier), we think between the jumps the BNS behaves too calmly to explain real market data of barrier options very well. This is probably due to the structure of the Ornstein-Uhlenbeck process modeling the stochastic volatility of the BNS model.

As indicated by U, the random paths generated by the Lévy models have quite a high fluctuations leading to a (too) high probability of hitting the barrier and therefore underestimating the prices of the barrier options. Lévy models with stochastic volatility (but without leverage) tend to overvalue down-and-out barrier options compared to the HES model, see Schoutens et al. (2003).

We think the high fluctuations of the models VG_CIRL, NIG_CIRL and MJD_CIRL are due to the direct linkage of the CIR process and the log stock price, i.e. the way a leverage is incorporated into the Lévy models is responsible of the high fluctuations of the random sample path and the relative low prices of barrier down-and-out options.

The HES model is a continuous model, it does not contain any jumps but the random sample paths generated by the HES model have moderately higher fluctuations compared to the HESJ model and measured by U, and the HES model undervalues the barrier options relative to the HESJ model slightly.

5 Conclusion

We test the performance of six different advanced stock price models for a given set of time series of market prices of European plain vanilla put and call options and barrier down-and-out call options for the period between 05/07/2017 and 21/08/2017 issued by different banks. At each timepoint and for each issuer we calibrate seven models to given prices of plain vanilla options. The models are:

the Heston model, see Heston (1993) and its generalization, see Bates (1996), the BNS model, see Barndorff-Nielsen and Shephard (2001), three Lévy models with stochastic time-change and leverage, see Carr et al. (2003) and a local volatility model.

All models can be fitted almost equally well to market data of plain vanilla options but computing the prices of the barrier options using the various calibrated models leads to significantly different prices for the exotic options. In particular, the BNS model overvalues barrier options by about 19% on average, the local volatility and the Heston model undervalue those options slightly and the Bates model reproduces barrier option prices very well.

Jessen and Poulsen (2013) worked with real market data of foreign-exchange barrier options and similarly concluded that the Heston model slightly undervalues barrier options, but the Bates model performs significantly worse than the Heston model, which stands in contrast to our results. Future research need to be done to explain why adding jumps to the Heston model increase the valuation ability of the model when applied to equity data and decrease the model performance for foreign-exchange barrier options.

Lévy models with stochastic time-change and leverage undervalue the exotic options by about 13% on average. There is barely any difference between the models NIG_CIRL and VG_CIRL. Compared to the other models, the model MJD_CIRL is the least robust one. Calibrating the MJD_CIRL model sometimes lead to unreasonably parameter sets. The results are similar for all issuers.

The findings that advanced stock prices models can be fitted very well to plain vanilla market data and the fact that those calibrated models predict very different prices for exotic options are in line with other studies in literature. In contrast to other studies, we are able to compare the simulated prices of exotic options to real market data and conclude that for the particular data set and period we looked at, the Heston model with jumps best explains barrier option market prices.

A heuristic analysis suggests that the different degree of fluctuation of the random paths of the models under the risk-neutral measure are responsible of producing different prices for the barrier options. The fluctuations are measured by the expected number of up-crossings of the stock price process of some interval. Further research need to be done in this direction.

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